# Financial Mathematics for Actuaries (Third Edition)

Chapter 9

Interest Rates and Financial Securities

# Learning Objectives

- Determination of the level of interest rates
- Yields and rates of interest of various financial securities
- The role of the Central Bank
- The Federal Reserve System of the U.S.
- Macroeconomic management and interest rates
- Interest rates in an open economy

#### 9.1 Interest Rate Determination

- Depending on the type of financial security, the rate of interest (yield) earned may be explicit (e.g., a savings account) or imputed from the security prices (e.g., Treasury bonds).
- Interest rate is an important policy variable for the government to manage its economy. It can be viewed as the *price* or *cost* of borrowing or acquiring capital.
- On the demand side, capital is needed when there is opportunity to invest.
- On the supply side, capital may depend on the following factors:
  - the time preference of consumers (willingness to give up current consumption for higher future consumption),

- the risk appetite of lenders (willingness to take risk),
- the expected rate of inflation (possible loss of purchasing power when consumption is deferred),
- the characteristics of the financial securities.
- As short-term Treasury securities (say, 90-day Treasury bill) are risk-free and highly liquid, we use the yield on these securities as the benchmark for determining the yields on other securities.
- The rate of return on short-term Treasury bill is called the **nominal** risk-free rate of interest.
- The term "nominal" refers to the fact that the rate of interest does not take into account inflation. It is also called the **money risk-free rate of interest** or **quoted risk-free rate of interest**. The

frequency of compounding (and hence the notion of effective rate of interest) has no bearing on its meaning here.

- Inflation refers to the increase in the general price level of goods and services.
- The **rate of inflation** is usually measured by the rate of increase of a general price index, such as the consumer price index (CPI), whole-sale price index or gross domestic product deflator.
- We define the **real rate of interest** as the interest earned by an investment after taking account of the erosion of purchasing power due to inflation.
- We denote  $r_I$  as the inflation rate, and  $r_N$  as the nominal rate of

interest. The real rate of interest  $r_R$  is then defined by the equation

$$1 + r_R = \frac{1 + r_N}{1 + r_I},\tag{9.1}$$

from which we obtain

$$r_R = \frac{1 + r_N}{1 + r_I} - 1 = \frac{r_N - r_I}{1 + r_I}.$$
 (9.2)

• If the rate of inflation  $r_I$  is low, we can write (9.2) as

$$r_R \approx r_N - r_I, \tag{9.3}$$

which says that the real rate of interest is approximately equal to the nominal rate of interest minus the rate of inflation.

• (9.1) and (9.3) are  $ex\ post$  relationships. They hold empirically.

• Irving Fisher argued that the nominal rate of interest *ought* to increase one for one with the *expected* rate of inflation, so that

$$r_N = r_R + \mathcal{E}(r_I), \tag{9.4}$$

where  $E(r_I)$  is the expected rate of inflation over the term of the financial asset, also called the **inflation premium**.

• Applying the Fisher equation to short-term Treasury bill, we have

Nominal risk-free rate of interest = real risk-free rate of interest + inflation premium. (9.5)

• While Treasury securities are risk free, other financial assets may be subject to default. Hence, investors may demand higher returns to compensate for the risk, and the rate of interest of the security would incorporate a **default risk premium**.

- For a corporate bond, the default risk can be measured by the bond's rating. A corporate bond with a higher rating will have a lower risk of default. Thus, its rate of interest will generally be lower.
- The liquidity of an asset also influences its interest rate. If the asset may not be bought or sold easily due to lack of liquidity, investors will demand compensation for this, creating a spread called the **liquidity premium**. As U.S. short-term Treasury securities are highly liquid, investors do not demand a liquidity premium.
- Long-term securities may also trade with a premium due to the uncertainty inherent to a longer maturity and holding period, called the **maturity risk premium** or **term premium**. The term premium is generally higher the longer the time to maturity of the asset.
- The nominal rate of interest of a financial asset may be generally

written as follows:

Nominal rate of interest = real risk-free rate of interest + inflation premium + default risk premium + liquidity premium + term premium. (9.6)

#### 9.2 Financial Securities

- The Federal funds and Discount Window credit are the main policy instruments through which the Federal Reserve Bank (the Fed) manages its monetary policy.
- Federal funds are the reserve balances that private banks keep at the Fed.
- Private banks lend Federal funds to each other to meet temporary shortages in liquidity.
- While the Fed cannot dictate the interest rate banks charge each other for such borrowing, it regularly announces its target rate to let market participants know about its policy direction.

- As of 30 October 2019, the target range for the **Federal Funds**Rate was 1.5% to 1.75%.
- When the Fed wants to increase reserves (and lower the Federal Funds Rate), it buys Treasury securities.
- When the Fed wants to reduce reserves (and raise the Federal Funds Rate) it sells Treasury securities.
- The short-term transactions the Fed uses for this purpose are called sale and repurchase agreements or repos.
- The **Discount Window** is an instrument that allows eligible institutions and banks to borrow money directly from the Fed on a short-term basis to meet temporary shortage of liquidity.

- Unlike the Federal funds rate, the Fed has direct control over the interest rate charged at the Discount Window.
- Commercial paper is a security issued by large corporations to raise funds to meet short-term financial needs. It is typically backed by a bank or the issuing corporation itself without any collateral.
- Bank prime loan rate is one of several base rates used by banks to price short-term commercial loans.
- corporate bond yields are the average yields of Moody's seasoned Aaa and Baa bonds, which are the ratings for "prime" and "lower medium" grade bonds, respectively.
- Eurodollars are time deposits in U.S. dollar in banks outside the U.S., and they are thus not under the jurisdiction of the Fed.

• London is currently the most important global market for trading currencies worldwide. The **London Interbank Offered Rate** (LIBOR) is the interest rate offered by leading banks in London that they would charge to other borrowing banks.

# 9.3 Inflation and Central Bank Policy

- The Federal Reserve System (the Fed) is the central banking system of the U.S. The **Federal Reserve Act** mandates the Fed with three duties: Sustain maximum employment, maintain stable prices and moderate long-term interest rates.
- The Fed has seven governors, who are appointed by the U.S. President.
- It is comprised of twelve regional Federal Reserve Banks.
- An important committee in the Fed is the **Federal Open Market**Committee (FOMC), which consists of the seven governors of the

  Fed, as well as the twelve presidents of the regional Federal Reserve

Banks. The FOMC is in charge of carrying out the monetary policies of the Fed.

- The Fed influences the rate of interest (and hence the rate of inflation) through controlling the amount of reserves commercial banks are required to hold.
- They have the following three instruments to use in carrying out their mandate: Reserve ratio, Discount Window lending and open market operation.
- The **reserve ratio** is the fractional reserve requirement that a depository institution needs to hold in the Federal Reserve Banks.
- Increasing the reserve ratio causes a contraction in the money supply and thus an increase in the rate of interest.

- The Fed has its own lending facility, namely, the Discount Window, through which they lend directly to the commercial banks that need to boost their reserve to meet short-term regulatory requirements.
- The Fed sets the rate of interest for such borrowing. This rate has generally been about 100 basis points above the target Federal funds rate. Commercial banks usually seek alternative borrowing from other banks before using the Discount Window.
- The Fed engages in **open market operations** through sale and repurchase agreements (repos).
- Under a repo the Fed buys U.S. Treasury securities, U.S. agency securities or mortgage-backed securities from a primary dealer, who would buy them back when the agreement matures.

• This operation increases reserves and injects liquidity into the market, thus lowering the rate of interest.

# 9.4 Macroeconomic Management

- Governments have the important mandate to sustain economic growth and maintain full employment. They can fulfill these goals by managing the aggregate demand in the economy.
- When the economy is weak, the government can increase aggregate demand by directly spending on public projects.
- Changing government expenditure or taxation is referred to as **fiscal** policy.
- An expansionary fiscal policy involves increasing government expenditure or reducing tax, or both.
- A contracting fiscal policy is the reverse: reducing government expenditure or increasing tax, or both.

- Aggregate demand can also be managed through the use of **mone-tary policy**.
- Central banks can increase money supply by purchasing treasury bonds or other securities in their open market operations.
- The increase in money supply injects liquidity into the economy and drives down the rate of interest. This stimulates consumers to spend and companies to invest.
- Unlike fiscal policy, monetary policy usually has broader effects on the economy. As monetary policy is not directly targeting certain sectors of the economy, its impacts are usually spread across the economy.

### 9.4 Rate of Interest in an Open Economy

- During the global financial crisis of 2008, which originated from the U.S. economy, the Fed implemented quantitative easying.
- It injected a large amount of liquidity into the U.S. economy, driving the rate of interest to unprecedentedly low level.
- The effects of quantitative easying, however, does not stay only in the U.S. economy. Much of the liquidity leaks out of the U.S., partly to chase higher returns elsewhere. This caused the rate of interest to be globally depressed.
- In sum, in a globally open economy, interest rates are not just influenced by its own conditions and government policies. It is also

dependent on how inter-related it is with respect to other countries, as well as what policies other countries are implementing.